

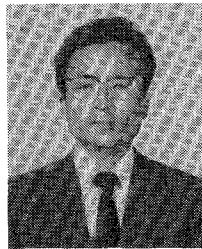
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Quasistatic Characteristics of Covered Coupled Microstrips on Anisotropic Substrates: Spectral and Variational Analysis

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Abstract — In this paper, expressions to compute the upper and lower bounds on true values of the even- and odd-mode capacitances of covered coupled microstrips over anisotropic substrates are obtained by using the Fourier transform and the variational approach. The method provides accurate calculation and yields the margins of error in the computation. Some examples are shown.

I. INTRODUCTION

IN RECENT YEARS, the boundary value problems involving microstrip lines on anisotropic substrates have been approached from numerical [1] and analytical points of view [2]-[9]. Alexopoulos *et al.* [2], [3] have shown the effect of an anisotropic substrate on the characteristics of covered coupled microstrips by using the method of moments. Methods for calculating the parameters of single [4], [5] and coupled microstrip lines [6]-[8] have been performed by applying transformation from anisotropic to isotropic problems. Green's functions for examples with anisotropic medium have been obtained using the image-coefficient method in [9].

The spectral-domain approach has been used extensively on problems of microstrip lines on isotropic substrates, and variational expressions of capacitances have been reported [10], [11]. This method was extended by [12] and [13] to analyze the characteristic parameters of single and coupled microstrips on anisotropic substrates.

The purpose of this paper is to solve the variational problem involving covered coupled microstrips on anisotropic substrates with an arbitrary permittivity tensor by using the Fourier transform, and obtaining in this way stationary expressions to compute the upper and lower bounds of the mode quasi-static characteristics of this structure. The method shows the equivalence between the mode capacitances of this structure and another with an isotropic substrate, in agreement with the reported results [14]. Besides, it is a fast and accurate calculation method in most practical cases and it yields the margins of error in the computation.

II. ANALYSIS

Consider the configuration of covered coupled microstrips shown in Fig. 1, which comprises two zero-thickness strips on an anisotropic dielectric substrate, which permit-

Manuscript received April 7, 1982; revised June 2, 1982.

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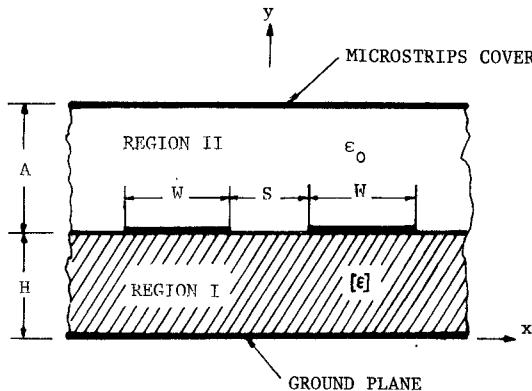


Fig. 1. Cross section of covered coupled microstrips.

tivity is given by the following tensor:

$$\bar{\epsilon} = \epsilon_0 \begin{bmatrix} \epsilon_{11}^* & \epsilon_{12}^* \\ \epsilon_{21}^* & \epsilon_{22}^* \end{bmatrix}. \quad (1)$$

One of these strips is at potential V and the other is at potential $\pm V$ for even or odd mode, respectively. Both, the ground and microstrips cover planes are at potential zero.

In the spectral-domain, a quasi-static solution to the potential problem can be obtained by solving Laplace's equation in the anisotropic region (Region I) as well as in the air region (Region II), subject to the proper boundary condition.

The potential distribution $\phi_1(x, y)$ satisfies the Laplace's equation in Region I

$$\nabla_T \cdot (\bar{\epsilon} \nabla_T \phi_1(x, y)) = 0. \quad (2)$$

After applying the boundary conditions, the Fourier transformed potential $\hat{\Phi}_1(\beta, y)$ can be expressed by

$$\hat{\Phi}_1(\beta, y) = \hat{V}_{e,o}(\beta) \frac{\sinh \left[|\beta| \left(\frac{\epsilon_{11}^*}{\epsilon_{22}^*} - \left(\frac{\epsilon_{12}^* + \epsilon_{21}^*}{2\epsilon_{22}^*} \right)^2 \right)^{1/2} \cdot y \right]}{\sinh \left[|\beta| \left(\frac{\epsilon_{11}^*}{\epsilon_{22}^*} - \left(\frac{\epsilon_{12}^* + \epsilon_{21}^*}{2\epsilon_{22}^*} \right)^2 \right)^{1/2} \cdot H \right]} \cdot \exp \left[-j\beta \frac{\epsilon_{21}^* + \epsilon_{12}^*}{2\epsilon_{22}^*} (y - H) \right] \quad (3)$$

where $\hat{V}_{e,o}(\beta)$ is the Fourier transformed potential on the air-anisotropic dielectric interface, even or odd mode.

In Region II, the solution of the Laplace's equation (spectral domain) $\hat{\Phi}_{II}(\beta, y)$ can be written

$$\hat{\Phi}_{II}(\beta, y) = \hat{V}_{e,o}(\beta) \frac{\sinh(|\beta|(A + H - y))}{\sinh(|\beta|A)}. \quad (4)$$

(3) and (4) can be used to obtain variational expressions for the upper and lower bounds on the capacitances per unit length $C_{e,o}$, even or odd-mode.

A. Upper Bound on $C_{e,o}$

The mode capacitances can be evaluated by the electric energy stored in this configuration $U_{e,o}$

$$C_{e,o} = \frac{U_{e,o}}{V^2}. \quad (5)$$

Now, we must calculate

$$U_{e,o} = U_I + U_{II}. \quad (6)$$

The electric energy stored in Region I U_I has been calculated in [12]

$$U_I = \frac{\epsilon_0}{2\pi} \int_0^\infty |\hat{V}_{e,o}(\beta)|^2 \beta \left(\epsilon_{11}^* \epsilon_{22}^* - \frac{(\epsilon_{12}^* + \epsilon_{21}^*)^2}{4} \right)^{1/2} \cdot \coth \left(\beta \left(\frac{\epsilon_{11}^*}{\epsilon_{22}^*} - \left(\frac{\epsilon_{12}^* + \epsilon_{21}^*}{2\epsilon_{22}^*} \right)^2 \right)^{1/2} \cdot H \right) d\beta \quad (7)$$

and the electric energy stored in Region II U_{II} can be evaluated using (7) with $\epsilon_{11}^* = \epsilon_{22}^* = 1$, $\epsilon_{12}^* = \epsilon_{21}^* = 0$, and $H \rightarrow A$.

Substituting in (5), the expression of $C_{e,o}$ is given by

$$C_{e,o} = \frac{\epsilon_0}{2\pi V} \int_0^\infty |\hat{V}_{e,o}(\beta)|^2 g(\beta) d\beta \quad (8a)$$

where

$$\hat{V}_e(\beta) = 2 \int_0^\infty V_e(x) \cos(\beta x) dx, \quad \text{even-mode} \quad (8b)$$

$$\hat{V}_o(\beta) = 2 \int_0^\infty V_o(x) \sin(\beta x) dx, \quad \text{odd-mode} \quad (8c)$$

and

$$g(\beta) = \beta \left(\coth(\beta A) + \left(\epsilon_{11}^* \epsilon_{22}^* - \frac{1}{4} (\epsilon_{12}^* + \epsilon_{21}^*)^2 \right)^{1/2} \cdot \coth \left(\beta \frac{\epsilon_{11}^*}{\epsilon_{22}^*} - \left(\frac{\epsilon_{12}^* + \epsilon_{21}^*}{2\epsilon_{22}^*} \right)^2 \right)^{1/2} \cdot H \right). \quad (9)$$

(8) is a variational expression for the mode capacitances of covered microstrips on anisotropic substrates similar to the one that was obtained in [12] for a single microstrip.

It is clear that assuming a suitable trial function for $V_{e,o}(x)$, the upper bounds of $C_{e,o}$ can be computed from (8). In this paper, the following expansion is proposed for $V_{e,o}(x)$ as an extension of the method reported in [11]–[13],

$$V_{e,o}(x) = \begin{cases} \sum_{i=1}^L a_i (d - x)^{-i}, & 0 \leq x \leq S/2 \\ 1, & S/2 \leq x \leq S/2 + W \\ \sum_{j=1}^{N+1} b_j (x - d)^{-j}, & x \geq S/2 + W \end{cases} \quad (10)$$

where

$$L = \begin{cases} M+1, & \text{even-mode} \\ M+2, & \text{odd-mode} \end{cases}$$

and

$$d = \frac{1}{2}(S + W).$$

Considering the $V_{e,o}(x)$ values at both strips edges and substituting the Fourier transform of (10) into (8a), we find that (8a) takes the form

$$\frac{C_{e,o}}{\epsilon_0} = \frac{2}{\pi} \int_0^\infty \left(\psi_0 + \sum_{i=1}^M a_i \psi_i + \sum_{j=1}^N b_j \Gamma_j \right)^2 g(\beta) d\beta \quad (11)$$

where ψ_0 , ψ_i , and Γ_j have been given in [13].

The coefficients a_i and b_j have been calculated using the Rayleigh-Ritz procedure. Hence, we have

$$\frac{C_{e,o}}{\epsilon_0} = \frac{2}{\pi} \left(\psi_{00} + \sum_{i=1}^M a_i \psi_{0i} + \sum_{j=1}^N b_j \Gamma_{0j} \right) \quad (12)$$

where a_i and b_j are obtained from the following system of equations:

$$\sum_{k=1}^M a_k \psi_{ik} + \sum_{j=1}^N b_j \xi_{ij} = -\psi_{0i}, \quad i = 1, \dots, M \quad (13a)$$

$$\sum_{i=1}^M a_i \xi_{ij} + \sum_{l=1}^N b_l \Gamma_{jl} = -\Gamma_{0j}, \quad j = 1, \dots, N \quad (13b)$$

and ψ_{00} , ψ_{0i} , Γ_{0j} , ψ_{ik} , ξ_{ij} , and Γ_{jl} have been given in [13].

By using (12), we can determine the mode capacitances (upper bound) for covered coupled microstrips on anisotropic substrates.

The analysis outlined above has been carried out for covered coupled microstrips, with strips width W_{eq} and separated by a distance S_{eq} over an isotropic substrate with relative permittivity ϵ_{eq}^* and thickness H_{eq} , and a separation between microstrips cover plane and air-substrate interface A_{eq} . The $C_{e,o}$ expressions for this structure are identical to those obtained by (12), giving

$$\epsilon_{eq}^* = \left(\epsilon_{11}^* \epsilon_{22}^* - \frac{1}{4} (\epsilon_{12}^* + \epsilon_{21}^*)^2 \right)^{1/2} \quad (14a)$$

$$H_{eq} = \left(\frac{\epsilon_{11}^*}{\epsilon_{22}^*} - \left(\frac{\epsilon_{12}^* + \epsilon_{21}^*}{2\epsilon_{22}^*} \right)^2 \right)^{1/2} \cdot H \quad (14b)$$

$$W_{eq} = W \quad (14c)$$

$$S_{eq} = S \quad (14d)$$

$$A_{eq} = A. \quad (14e)$$

Consequently, the mode capacitances of covered coupled microstrips on anisotropic substrates are equivalent to a similar structure on an isotropic substrate with relative permittivity and thickness given by (14a) and (14b), respectively.

For practical anisotropic dielectric substrates $\epsilon_{12}^* = \epsilon_{21}^*$, if a steady magnetic field is not applied to the dielectric [15], ϵ_{eq}^* and H_{eq} are given by

$$\epsilon_{eq}^* = \left(\epsilon_{11}^* \epsilon_{22}^* - (\epsilon_{12}^*)^2 \right)^{1/2} \quad (15a)$$

$$H_{eq} = \left(\frac{\epsilon_{11}^*}{\epsilon_{22}^*} - \left(\frac{\epsilon_{12}^*}{\epsilon_{22}^*} \right)^2 \right)^{1/2} \cdot H. \quad (15b)$$

These are in agreement with other reported results [14].

B. Lower Bound on $C_{e,o}$

The stationary values expression of the capacitance per unit length given in [10] can be used to obtain the lower bounds on the mode capacitances of the covered coupled microstrips over arbitrary anisotropic substrate. Therefore, we apply the transformation equations (14) to that expres-

sion and readily obtain

$$\frac{C_{e,o}}{\epsilon_0} = \frac{1}{2\pi Q^2} \int_0^\infty G(\beta) |\hat{\rho}_{e,o}|^2 d\beta \quad (16)$$

where Q is the charge on one strip

$$\hat{\rho}_e(\beta) = 2 \int_{S/2}^{S/2+W} \rho_e(x) \cos(\beta x) dx, \quad \text{even-mode} \quad (17a)$$

$$\hat{\rho}_o(\beta) = 2 \int_{S/2}^{S/2+W} \rho_o(x) \sin(\beta x) dx, \quad \text{odd-mode} \quad (17b)$$

and

$$G(\beta) = (\beta (\coth(\beta A) + \epsilon_{eq}^* \coth(\beta H_{eq})))^{-1} \quad (18)$$

Following a similar procedure to the one used in upper bound calculation, ρ_e and ρ_o are expended in terms of a power series of X , $X = x - S/2$

$$\rho_e = \sum_{i=1}^{M+1} a_i X^{i-1} \quad (19a)$$

$$\rho_o = \sum_{i=1}^{M+1} a_i (W - X)^{i-1}. \quad (19b)$$

a_{M+1} can be obtained from Q and the remaining a_i using the following system of equations

$$\sum_{j=1}^M a_j \psi_{ij} = -\psi_{0i}, \quad i = 1, \dots, M. \quad (20)$$

Therefore, we have

$$\frac{C_{e,o}}{\epsilon_0} = \frac{\pi}{2} \left(\psi_{00} + \sum_{i=1}^M a_i \psi_{0i} \right)^{-1} \quad (21)$$

where ψ_{00} , ψ_{0i} , and ψ_{ij} have been given in [13].

Both (12) and (21) are useful to compute the mode capacitances, upper and lower bounds, for covered coupled microstrips on anisotropic substrates, so that the margins of error in the calculation can be known. The accuracy of the method is insured by increasing M and N in (12) and M in (21).

III. EXAMPLES

As an application of the method outlined above, the quasi-static characteristics of some particular configurations have been computed. In these cases, the mode impedances ($Z_{e,o}$, even or odd mode) and normalized phase velocities ($v_{e,o}/c$, even or odd mode) can be calculated by the following relations:

$$Z_{e,o} = \frac{1}{c} (C_{e,o} C_{e,o}^v)^{-1/2} \quad (22a)$$

$$\frac{v_{e,o}}{c} = \left(\frac{C_{e,o}^v}{C_{e,o}} \right)^{1/2} \quad (22b)$$

where $C_{e,o}^v$ denotes the mode capacitances with the substrate removed.

First, we have computed $Z_{e,o}$ for covered coupled micro-

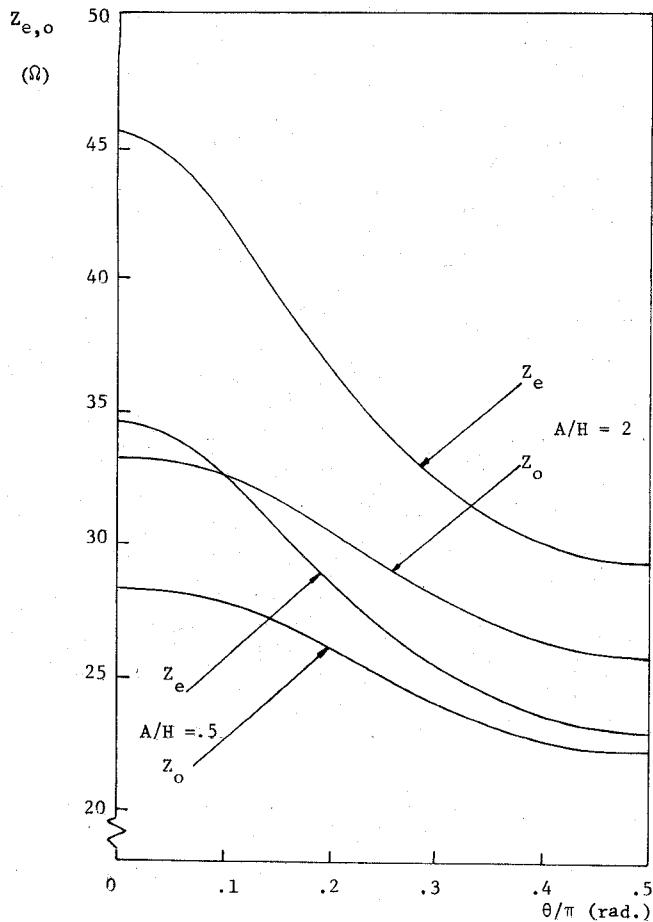


Fig. 2. Even- and odd-mode impedances Z_e and Z_o versus θ for covered coupled microstrips on anisotropic substrates ($\epsilon_{\xi}^* = 40$, $\epsilon_{\eta}^* = 10$, $W/H = S/H = 1$).

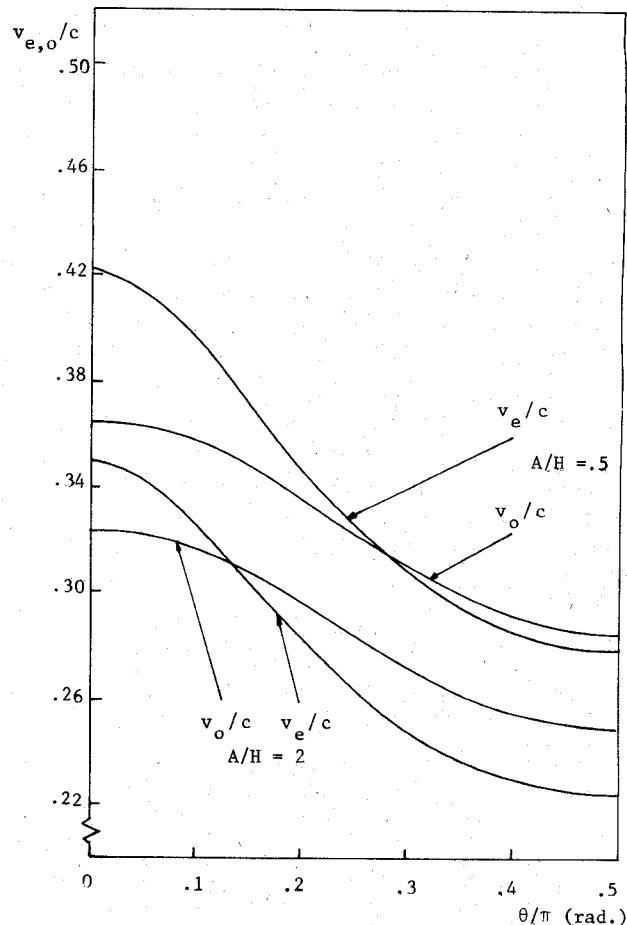


Fig. 3. Even- and odd-mode normalized phase velocities v_e/c and v_o/c versus θ for covered coupled microstrips on anisotropic substrates ($\epsilon_{\xi}^* = 40$, $\epsilon_{\eta}^* = 10$, $W/H = S/H = 1$).

strips on isotropic substrates from the average values of the upper and lower bounds of $C_{e,o}$ (12) with $M = 2$ and $N = 1$, and (21) with $M = 2$, respectively. It is found that the calculated impedances using this procedure are very accurate when compared with the reported in [16].

$C_{e,o}^+$ and $C_{e,o}^-$ for a single-crystal sapphire substrate ($\epsilon_{11}^* = 9.40$, $\epsilon_{22}^* = 11.60$, $\epsilon_{12}^* = \epsilon_{21}^* = 0$, [1]) have been computed using (12), upper bound on $C_{e,o}(C_{e,o}^+)$, with $M = 2$ and $N = 1$, and (21), lower bound on $C_{e,o}(C_{e,o}^-)$, with $M = 2$. The results are summarized in Table I. The margin of absolute error in the calculation can be estimated practically between 2 and 4 percent for even-mode and between 4 and 5.5 percent for odd-mode, over range of A/H , S/H , and W/H which were calculated with the previous values of M and N . When a smaller difference between the upper and lower bounds on $C_{e,o}$ is required, or the strip edges singularities were significative, the computed values can be improved by increasing M and N .

Finally, an anisotropic substrate cut in the direction with an angle θ from the principal axis ($\xi - \eta$ axis) has been also considered. Figs. 2 and 3 show the mode impedances and normalized phase velocities, respectively, as functions of θ for $\epsilon_{\xi}^* = 40$, $\epsilon_{\eta}^* = 10$, $W/H = 1$, $S/H = 1$, $A/H = 0.5$, 2, where ϵ_{ξ}^* and ϵ_{η}^* are the relative permittivities along the ξ and η directions, respectively. It can be shown that $v_e = v_o$

TABLE I
MODE CAPACITANCES OF COVERED COUPLED MICROSTRIP LINES

A/H	S/H	W/H	SAPPHIRE SUBSTRATE $\epsilon_{11}^* = 9.40$, $\epsilon_{22}^* = 11.60$						WITHOUT SUBSTRATE			
			C_e^+ ϵ_{ξ}^*	C_e^- ϵ_{ξ}^*	C_o^+ ϵ_{ξ}^*	C_o^- ϵ_{ξ}^*	C_e^+ ϵ_{η}^*	C_e^- ϵ_{η}^*	C_o^+ ϵ_{η}^*	C_o^- ϵ_{η}^*		
0.5	0.5	1.0	21.54	21.02	4.51	4.38	27.90	26.69	5.42	5.18		
		2.0	35.69	34.37	7.67	7.33	42.06	39.91	8.59	8.12		
1.0	1.0	1.0	23.05	22.32	4.77	4.60	25.58	24.50	5.09	4.86		
		2.0	37.17	35.65	7.90	7.56	39.80	37.71	8.27	7.79		
1.0	0.5	1.0	20.34	19.85	3.32	3.24	27.07	25.89	4.60	4.40		
		2.0	33.37	32.22	5.37	5.20	40.13	37.98	6.68	6.32		
2.0	1.0	1.0	21.83	21.16	3.57	3.47	24.62	23.58	4.14	3.96		
		2.0	34.86	33.50	5.63	5.43	37.74	35.82	6.21	5.91		
2.0	0.5	1.0	19.82	19.34	2.77	2.69	26.85	25.67	4.36	4.16		
		2.0	32.29	31.23	4.27	4.17	39.50	37.33	5.97	5.66		
2.0	1.0	1.0	21.28	20.63	2.99	2.90	24.34	23.31	3.83	3.67		
		2.0	33.77	32.51	4.50	4.38	36.96	35.12	5.42	5.18		

for $\theta = 50.8^\circ$ when $A/H = 0.5$, and $\theta = 24.1^\circ$ if $A/H = 2$. For open coupled-strips case ($A \rightarrow \infty$), $v_e = v_o$ for $\theta = 11.9^\circ$, which is good agreement with the results [8]. The previous results were obtained by (21), $M = 2$, and the relations (22).

IV. CONCLUSIONS

In this paper, we have shown that the Fourier transform and the variational approach are useful to obtain two stationary expressions for the mode capacitances of covered coupled microstrip lines on an arbitrary anisotropic substrate. Both expressions provide a fast and accurate calculation of the upper and lower bounds on the mode capacitances. Besides, they yield the margins of error in the computation. The presented analysis shows the equivalence between the mode capacitances of these lines and others with an isotropic substrate. As examples of applications, upper and lower bounds on mode capacitances of covered coupled microstrips with sapphire substrate and others without substrate are computed. Finally, it is shown in graphical form that the phase velocities of these structures can be matched if an anisotropic substrate is used, in agreement with reported results for open coupled microstrips [8].

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